

Real Number

Prime no.(s) :- No.(s) having 2 factors (1 and itself).

Composite no(s) :- No.(s) having more than 2 factors.

☆ Every Composite no. can be expressed as a product of prime and this factorization is unique.

H.C.F (a, b) → Sabse bada common factor.

L.C.M (a, b) → sabse chotta no. jo dono se divide ho jaye.

☆ $H.C.F(a, b) \times L.C.M(a, b) = a \times b$

☆ Rational no. = $\frac{p}{q}$ > Co-prime no.(s) and $q \neq 0$

• Co-prime no.(s) are no.(s) having only '1' as common factor.

H.C.F of co-prime no.(s) = 1

• let x be a prime no.

if x divides y^2 then x divides y .
Where y is a positive integer.

My Top Ten

- Two numbers are in the ratio $2:3$ and the LCM is 180. What is the HCF of these numbers? [CBSE 2023]
- If $\text{HCF}(26, 169) = 13$, the $\text{LCM}(26, 169) = ?$
(a) 26 (b) 52 (c) 338 (d) 13
- Find by prime factorisation the LCM of the numbers 18180 and 7575. Also, find the HCF of the two numbers. [CBSE 2023]
- If two positive integers m and n expressible in the form $m = pq^3$ and $n = p^3q^2$, where p, q are prime numbers, then $\text{HCF}(m, n) =$
(a) pq (b) pq^2 (c) p^3q^3
- The sum of two positive numbers is 240 and their HCF is 15. Find the number of pairs of numbers satisfying the given condition.
- Find the largest positive integer that will divide 398, 436 and 542 leaving remainders 7, 11, and 15 respectively.
- There is a circular path around a sports field. Priya takes 18 minutes to drive one round of the field, while Ravish takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point? [NCERT]
- In a school there are two sections — section A and section B of class X. There are 32 students in section A and 36 students in section B. Determine the minimum number of books required for their class library, so that they can be distributed equally among students of section A or section B.
- Prove that $\sqrt{2}$ is an irrational number. [NCERT, CBSE 2010]
- Show that $5 - 2\sqrt{3}$ is an irrational number. [CBSE 2009]

polynomial

POLYNOMIALS :- Algebraic expression where power of variable is non-negative integer.
Many terms whole no.

- ★ Exponent of highest degree term in polynomial is called **Degree**.
- ★ If $P(x)$ is a polynomial,
 - value of $P(x)$ at $x = \alpha$ is obtained by replacing x with ' α ' in whole expression
 - ' α ' is said to be zero of polynomial if $P(\alpha) = 0$
- ★ Graphically,
Zeros of polynomial are the x -coordinate of the point where graph intersects x -axis.
- ★ If α, β are the zeros of polynomial.

$$P(x) = ax^2 + bx + c$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

- ★ let zeros are α, β
Quadratic polynomial is given by

$$K(x^2 - (\alpha + \beta)x + \alpha\beta)$$

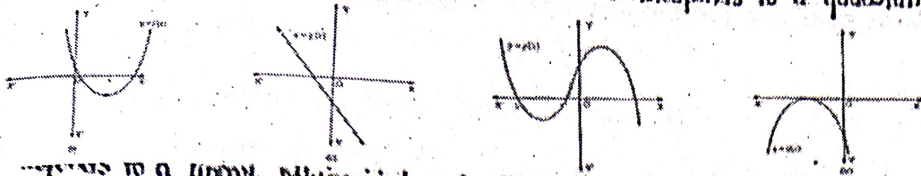
- ★ General formula

$$K(x^2 - (\text{sum of zeros})x + \text{product of zeros})$$



My Top Ten

1. If each one of the following graphs in the graph of a polynomial, then identify which one corresponds to a linear polynomial and which one corresponds to a quadratic polynomial?



2. If α and β are the zeros of quadratic polynomial $f(x) = x^2 - px + q$, then find the values of (i) $\alpha^2 + \beta^2$ (ii) $\frac{1}{\alpha} + \frac{1}{\beta}$

3. If α, β are the zeros of the polynomial $f(x) = 2x^2 + 5x + k$ satisfying the relation $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k for this to be possible.

4. If α and β are the zeros of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$, find a polynomial whose zeros are $2\alpha + 3\beta$ and $3\alpha + 2\beta$.

5. If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 5x + 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$.

6. If the sum of the zeros of the quadratic polynomial $f(t) = kt^2 + 2t + 3k$ is equal to their product, find the value of k .

7. If the squared differences of the zeros of the quadratic polynomial $f(x) = x^2 + px + 45$ is equal to 144, find the value of p .

8. Assertion (A): $(2 - \sqrt{3})$ is one zero of the quadratic polynomial then other zero will be $(2 + \sqrt{3})$.

Reason (R): Irrational zeros always occurs in pairs.

(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, and Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

9. The graph of a quadratic polynomial $p(x)$ passes through the points $(-6, 0)$, $(0, -30)$, $(4, 20)$ and $(6, 0)$. The zeroes of the polynomial are

- (a) $-6, 0$ (b) $4, 6$ (c) $-30, 20$ (d) $-8, 9$

10. In a pool at an aquarium, a dolphin jumps out of the water travelling at 20 cm per second. Its height above water level after t seconds is given by $h = 20t - 16t^2$. [CBSE 2023]



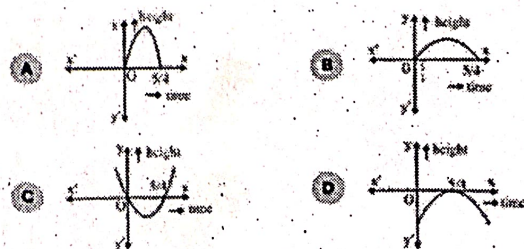
Based on the above, answer the following questions:

(i) Find the zeroes of polynomial $p(t) = 20t - 16t^2$.

(ii) Which of the following type of graph represents $p(t)$?

(iii) What would be the value of h at $t = \frac{3}{2}$? Interpret the result.

(iv) How much distance has the dolphin covered before hitting the water level again?



Pair of linear Equations in 2 variables

★ Pair of linear equation in two variable x & y can be represented by

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Pair of linear Equation can be solved by

Graphical method

★ if pair of linear eqⁿ intersect at a point, then pair of linear eqⁿ is having unique solution.

★ if pair of linear eqⁿ are parallel then pair has no solution, i.e inconsistent.

★ if pair of linear equation is coincident, then it has infinity many solution.

$$\begin{aligned} \star a_1x + b_1y + c_1 &= 0 \\ a_2x + b_2y + c_2 &= 0 \end{aligned}$$

$$\star \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \longrightarrow \text{unique solution}$$

$$\star \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \longrightarrow \text{no solution, inconsistent}$$

$$\star \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \longrightarrow \text{Infinity many solution}$$

Algebraic method

- ✓ substitution method
- ✓ Elimination method
- ✓ cross multiplication method.

My Top Ten

1. Solve the following system of linear equations graphically: $x - y = 1$ $2x + y = 8$ **Ans:** (3,2), 13 S sq units
 solve the area bounded by these two lines and y-axis. Also, determine this area.

2. solve the following systems of equations by using the method of substitution:

(i) $2x + 3y = 9$
 $3x + 4y = 5$

(ii) $\frac{2x}{a} + \frac{y}{b} = 2$

Ans: (1) $x = -21, y = 17$ (2) $x = 2a, y = -2b$

$\frac{x}{a} - \frac{y}{b} = 4$

3. Solve: $37x + 41y = 70$
 $41x + 37y = 86$ **Ans:** $x = 3, y = -1$

4. For each of the following systems of equations determine the value of K for which the given system of equations has a unique solution: **Ans:** (1) all real values of k except $2/3$
 (2) all values of K except $10/3$

(i) $x - Ky = 2$

(ii) $2x - 3y = 1$

$3x + 2y = -5$

$Kx + 5y = 7$

5. For each of the following systems of equations determine the value of K for which the given system of equations has infinitely many solutions: **Ans:** (1) $k = 3/2$ (2) $k = 6$

(i) $5x + 2y = K$

(ii) $(K-3)x + 3y = K$

$10x + 4y = 3$

$Kx + Ky = 12$

→ A two digit number is obtained by either multiplying sum of the digits by 8 and adding 1 or by multiplying the difference of the digits by 13 and adding 2. Find the number. **Ans:** 41

→ Five years hence, father's age will be three times the age of his son. Five years ago, father was seven times as old as his son. Find their present ages. **[NCERT]**
Ans: father = 40 years, son = 10 years

→ the sum of a two digit number & the number obtained by reversing the order of its digit is 121, & the two digits differ by 3. find the number. **Ans:** 47 or 74

→ A part of monthly hostel charges in a college are fixed and the remaining depend on the number of days one has taken food in the mess. When a student A takes food for 20 days, he has to pay ₹ 1000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹ 1180 as hostel charges. Find the fixed charge and the cost of food per day. **[NCERT]**
Ans: Fixed charge = 400 rs, cost of food per day = 30 rs

→ For each of the following system of equations determine the values of K for which the given system has no solution:

(i) $3x - 4y + 7 = 0$

(ii) $2x - Ky + 3 = 0$

$Kx + 3y - 5 = 0$

$3x + 2y - 1 = 0$

Ans: (1) $k = -9/4$ (2) $k = -4/3$

Quadratic Equations

★ If $P(x) = ax^2 + bx + c$ is a quadratic polynomial then $P(x) = 0$ i.e. $ax^2 + bx + c = 0$ $a \neq 0$ is called **Quadratic Equation**.

★ A real number ' α ' is said to be root of equation $ax^2 + bx + c = 0$ if

- $a\alpha^2 + b\alpha + c = 0$

- ' α ' is the zero of polynomial

$$P(x) = ax^2 + bx + c$$

★ Solution of Quad. equation.

$$ax^2 + bx + c = 0, a \neq 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$$

$$\text{Discriminant} = \Delta = b^2 - 4ac$$

★ $\checkmark D > 0$ (+ve) \rightarrow Real and Distinct roots

$\checkmark D = 0$ \rightarrow Real and equal roots

$\checkmark D < 0$ (-ve) \rightarrow Non-real roots

Arithmetic progression

Sequence: Arrangement of numbers in a definite order according to some rule.

Arithmetic Sequence: Sequence $a_1, a_2, a_3, \dots, a_n$ is called A.P.
if $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \dots = a_{n+1} - a_n = d$

$a_{n+1} - a_n$ is independent of n .

$a_1 =$ first term (a)

$d =$ common difference

Type of Arithmetic Progression

Finite A.P.

A.P. containing a finite no. of terms \rightarrow finite A.P.

e.g. 3, 5, 7, 9, 11, 13, ...

Infinite A.P.

A.P. which does not have a finite no. of terms

e.g. 10, 20, 30, 40, 50, ...

General/ n^{th} term:

$$T_n = a + (n-1)d$$

first term

common difference

sum of n terms of an A.P.:

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{n}{2} [a + T_n]$$

My Top Ten

1. The sum of first m terms of an A.P. is $4m^2 - m$. If its n th term is 107, find the value of n . Also, find the 21st term of this A.P. [CBSE 2013]
Ans: $n=1, a_{21} = 163$
2. If an A.P., the sum of first ten terms is -150 and the sum of its next ten terms is -550. Find the A.P. [CBSE 2010]
Ans: $a = 3, d = -4$
3. The sum of 1th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th term is 34. Find the first term and the common difference of the A.P. [NCERT] Ans: $-1/2, 5/2$
4. The sum of first n terms of an A.P. is $5n^2 + 3n$. If its m th term is 168, find the value of m . Also, find the 20th term of this A.P. [CBSE 2013] Ans: $m=17, a_{20} = 198$
5. The first and the last terms of an A.P. are 7 and 49 respectively. If sum of all its terms is 420, find its common difference. [CBSE 2014] Ans: 3
6. How many terms of the series 54, 51, 48, ... be taken so that their sum is 513? Explain the double answer. [CBSE 2005]
Ans: $n = 18$ or 19
7. If an A.P. the sum of m terms is equal to n and the sum of n terms is equal to m , then prove that the sum of $(m+n)$ terms is $-(m+n)$. [proof question]
8. The ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$. Show that the ratio of the m th and n th terms is $(2m-1) : (2n-1)$. [CBSE 2016, 2017]
[proof question]

The sum of the first q terms of an A.P. is 162. The ratio of its 6th term to its 13th term is 1:2. Find the first and 15th term of the A.P. [CBSE 2015]

Ans: 6, 48

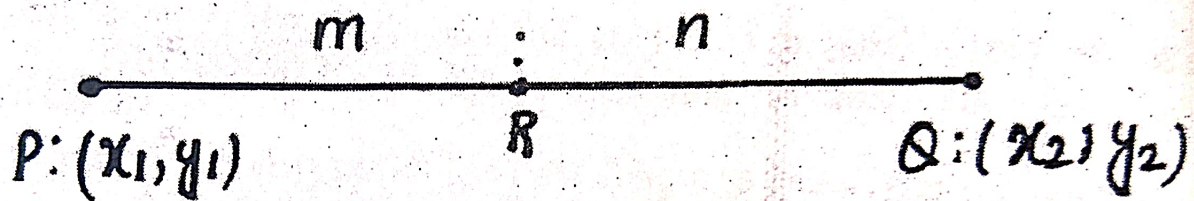
The sum of the first q terms of an A.P. is $63q - 3q^2$. If its p th term is -60 find the value of p . Also, find the 11th term of this A.P. [CBSE 2013]

Ans: $p = 21, a_{11} = 0$

Coordinate Geometry

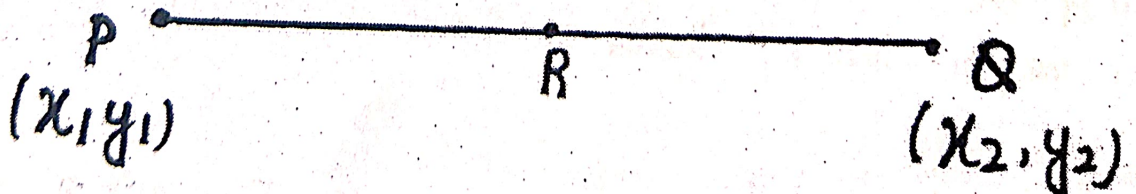
Distance b/w points $P: (x_1, y_1)$ & $Q: (x_2, y_2)$ is

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



$$PR : QR = m : n$$

$$R: \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$



if R is midpoint of PQ

$$\text{coordinate of } R: \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

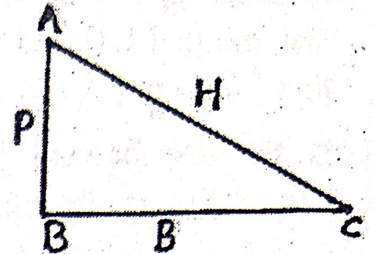
My Top Ten

- The line segment joining the points $(3, -4)$ and $(1, 2)$ is bisected at the points P and Q. If the coordinates of P and Q are $(p, -2)$ and $(5/3, q)$ respectively. Find the values of p and q. [CBSE 2005] Ans: $p=7/3, q=0$
- The line joining the points $(2, 1)$ and $(5, -8)$ is bisected at the points P and Q. If point P lies on the line $2x - y + k = 0$. Find the value of k. [CBSE 2008] Ans: $k=-8$
- A point P divides the line segment joining the points A $(3, -5)$, B $(-4, 8)$ such that $\frac{AP}{PB} = \frac{k}{1}$. If P lies on the line $x + y = 0$, then find the values of k. [CBSE 2012] Ans: $1/2$
- The line segment joining the points P $(3, 3)$ and Q $(6, -6)$ is bisected at the points A and B such that A is nearest to P. If A also lies on the line given by $2x + y + k = 0$, find the value of k. [CBSE 2009] Ans: -8
- The mid-point P of the line segment joining the points A $(-10, 4)$ and B $(-2, 0)$ lies on the line segment joining the points C $(-9, -4)$ and D $(-4, 4)$. Find the ratio in which P divides CB. Also, find the value of y. [CBSE 2014] Ans: $3:2, y=6$
- Show that $\triangle ABC$, where A $(-2, 0)$, B $(2, 0)$, C $(0, 2)$ and $\triangle PQR$, where P $(-4, 0)$, Q $(4, 0)$, R $(0, 4)$ are similar. [CBSE 2017] proof question
- If the points A $(a, -11)$, B $(5, b)$, C $(2, 15)$ and D $(1, 1)$ are the vertices of a parallelogram ABCD, find the values of a and b. Ans: $a=4, b=3$
- Find the ratio in which P $(4, m)$ divides the line segment joining the points A $(2, 3)$ and B $(6, -3)$. Hence, find m. [CBSE 2018] Ans: $1:1, m=0$
- If the points P, Q $(x, 7)$, R, S $(6, y)$ in this order divide the line segment joining A $(2, p)$ and B $(7, 10)$ in 5 equal parts, find x, y and p. [CBSE 2015] Ans: $x=4, y=9$
- Find the ratio in which the line segment joining $(-2, -3)$ and $(5, 6)$ is divided by (i) x-axis (ii) y-axis. Also, find the coordinates of the point of division in each case. [CBSE 2013] Ans: $2:5, (0, -3/7)$

Introduction Of Trigonometry

TRIGONOMETRY :-

$$\begin{aligned} \sin \theta &= P/H & \operatorname{cosec} \theta &= H/P \\ \cos \theta &= B/H & \sec \theta &= H/B \\ \tan \theta &= P/B & \cot \theta &= B/P \end{aligned}$$



IDENTITIES :-

$$\sqrt{\sin^2 \theta + \cos^2 \theta = 1} \quad \left\{ \frac{P^2}{H^2} + \frac{B^2}{H^2} = \frac{P^2 + B^2}{H^2} = \frac{H^2}{H^2} \right\}$$

$$\sqrt{1 + \cot^2 \theta = \operatorname{cosec}^2 \theta} \quad \left\{ 1 + \frac{B^2}{P^2} = \frac{P^2 + B^2}{P^2} = \frac{H^2}{P^2} \right\}$$

$$\sqrt{1 + \tan^2 \theta = \sec^2 \theta} \quad \left\{ 1 + \frac{P^2}{B^2} = \frac{B^2 + P^2}{B^2} = \frac{H^2}{B^2} \right\}$$

$$\star \operatorname{cosec} \theta = \frac{1}{\sin \theta}, \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\star \tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

TRIGONOMETRIC RATIO OF SOME SPECIFIC ANGLES

θ	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\cot \theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\operatorname{cosec} \theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

My Top Ten

1. If $4 \cot^2 - 45^\circ - \sec^2 60^\circ + \sin^2 60^\circ + p = 3/4$, find the value of p . [CBSE 2023]

2. Prove the following identities:

$$\frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta \quad \text{[CBSE 2000, 23]}$$

3. Prove the following identities:

$$(\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2 \theta + \cot^2 \theta \quad \text{[CBSE 2000, 19]}$$

4. Prove the following identities:

$$(1 + \cot \theta - \operatorname{cosec} \theta)(1 + \tan \theta + \sec \theta) = 2 \quad \text{[CBSE 2008, 19]}$$

5. Prove the following identities:

$$\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} = 1 + \tan A + \cot A = 1 + \sec A \operatorname{cosec} A \quad \text{[CBSE 2002, 19, 23]}$$

6. If $a \cos \theta + b \sin \theta = m$ and $a \sin \theta - b \cos \theta = n$, prove that $a^2 + b^2 = m^2 + n^2$ [CBSE 2001C, 2023]

7. Prove the following identities:

$$\frac{\tan \theta + \sec \theta - 1}{\tan \theta + \sec \theta + 1} = \frac{1 + \sin \theta}{\cos \theta} \quad \text{[CBSE 2002, 23]}$$

8. Prove the following identities:

$$(\operatorname{cosec} \theta - \sin \theta)(\sec \theta - \cos \theta) = \frac{1}{\tan \theta + \cot \theta} \quad \text{[CBSE 2000, 23]}$$

9. If $\sin \theta + \cos \theta = \sqrt{3}$, then find the value of $\sin \theta \cos \theta$. [CBSE 2023]

10. Prove the following identities:

$$\frac{1}{\sec A + \tan A} - \frac{1}{\cos A} = \frac{1}{\cos A} - \frac{1}{\sec A - \tan A} \quad \text{[CBSE 2023]}$$

Applications of Trigonometry

SOME APPLICATION OF TRIGONOMETRY

HEIGHT AND DISTANCE

LINE OF SIGHT:- It is the line down from the eye of the observer to the point in the object viewed by the observer.

In the fig. AC is the line of sight.

ANGLE OF ELEVATION:- It is the angle formed by the line of sight with the horizontal when object is below the point of observation.

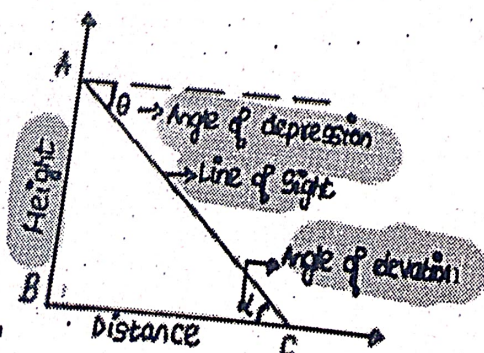
In the figure α is the angle of elevation, as C is the observer and A is the object.

ANGLE OF DEPRESSION:- It is the angle formed by the line of sight with the horizontal when object is below the point of observation.

In the figure B is the angle of depression, as A is the observer and C is the object.

HEIGHT AND DISTANCE:- In the fig. AB is height and BC is the distance.

$$\text{In } \triangle ABC, \quad \sin \alpha = \frac{AB}{AC}, \quad \cos \alpha = \frac{BC}{AC}, \quad \tan \alpha = \frac{AB}{BC}$$
$$\csc \alpha = \frac{AC}{AB}, \quad \sec \alpha = \frac{AC}{BC}, \quad \cot \alpha = \frac{BC}{AB}$$

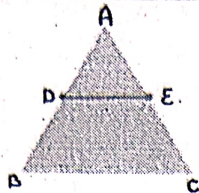


My Top Five

1. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is 30° . Find the height of the tower. [NCERT, CBSE 2023]
2. A vertical tower stands on a horizontal plane is surmounted by a vertical flag-staff of height 5 metres. At a point on the plane, the angles of elevation of the bottom and the top of the flag-staff are respectively 30° and 60° . Find the height of the tower. [CBSE 2015, 16, 19, 20]
3. The angle of elevation of the top of a tower from a point A on the ground is 30° . On moving a distance of 20 metres towards the foot of the tower to a point B, the angle of elevation increases to 60° . Find the height of the tower and the distance of the tower from the point A. [CBSE 2002, 15, 17]
4. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45° . Determine the height of the tower. [NCERT, CBSE 2014, 17, 23]
5. As observed from the top of a 75 m tall lighthouse, the angles of depression of two ships are 30° and 45° . If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. [NCERT, CBSE 2013, 23]

Triangles

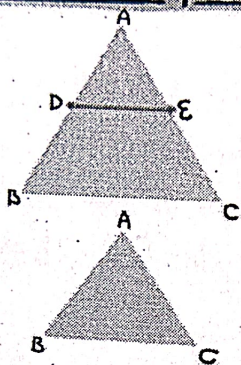
B.P.T / Thales Theorem :-



If $DE \parallel BC$
then

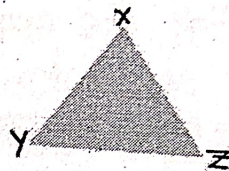
$$\frac{AD}{DB} = \frac{AE}{EC}$$

Converse of B.P.T :-



If $\frac{AD}{DB} = \frac{AE}{EC}$

$DE \parallel BC$



If $AB = AC$
then $\angle C = \angle B$

If $\angle Y = \angle Z$
then $XY = XZ$

* Angles opposite to equal sides are same.

* Sides opposite to equal angles are same.

⊕

congruent

shape size
same

similar

shape same

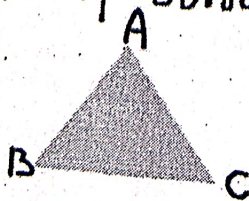
size may or may not be same.

similar polygons :-

✓ corresponding angles are equal.

✓ length of corresponding sides are proportional.

⊕

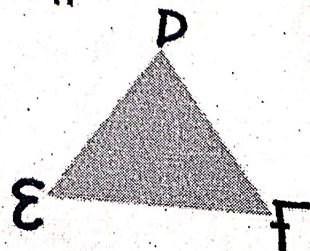


$\triangle ABC \sim \triangle DEF$

$$\angle A = \angle D$$

$$\angle B = \angle E$$

$$\angle C = \angle F$$



$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

My Top Ten

1. In fig. E is a point on the side CB produced of an isosceles triangle, ABC with $AB = AC$. If $AB \perp BC$ and $EF \perp AC$, prove that (i) $\triangle ABB \sim \triangle ECF$ (ii) $AB \times EF = AB \times EC$.

[NCERT, CBSE 2010]

2. D is a point on the side BC of $\triangle ABC$ such that $\angle ABC = \angle BAC$. Prove that $\frac{CA}{CB} = \frac{CB}{CA}$ or, $CA^2 = CB \times CD$. [NCERT, CBSE 2004]

3. Any point X inside $\triangle DEF$ is joined to its vertices. From a point P in DX, PQ is drawn parallel to DE meeting XE at Q and QR is drawn parallel to EF meeting XF in R. Prove that $PR \parallel DF$. [NCERT, CBSE 2002]

4. In fig., if $DE \parallel AG$ and $DF \parallel AR$. Prove that $EF \parallel QR$. [NCERT, CBSE 2008]

5. In fig., A, B and C are points on OP, OQ and OR respectively such that $AB \parallel PQ$ and $BC \parallel QR$. Show that $AC \parallel PR$. [NCERT]

6. In fig., $DE \parallel BC$ and $CD \parallel EF$. Prove that $AD^2 = AB \times AF$. [CBSE 2007]

7. In fig., we have $AB \parallel CD \parallel EF$. If $AB = 6$ cm, $CD = x$ cm, $EF = 10$ cm, $BD = 4$ cm and $DE = y$ cm, calculate the values of x and y . [Ans: $x = 3.75$ cm, $y = 6.67$ cm]

8. In fig., $\angle A = \angle CED$, prove that $\triangle CAB \sim \triangle CED$. Also, find the value of x . [Ans: 6]

9. In fig., if $AB \perp BC$, $DC \perp BC$ and $DE \perp AC$, prove that $\triangle CED \sim \triangle ABC$.

10. Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC in L and AB produced in E. Prove that $EL = 2BL$. [CBSE 2009, 23]

Probability

PROBABILITY:- it is the study of mathematics which calculates degree of uncertainty.

Experimental or Empirical probability:- Experimental probability is based on actual experiments and adequate recordings of the happening of events.

Mathematically,

$$P(E) = \frac{\text{Number of trials in which the event happened}}{\text{Total number of trials}}$$

Classical probability:-

classical probability is predication of results without performing these experiments actually.

$$P(E) = \frac{\text{Number of outcomes favorable to E}}{\text{Number of all possible outcomes of experiment}}$$

Elementary Event:- An event having only one outcomes of the experiment is called elementary event.

Note:- Sum of probabilities of all the elementary events of an experiment is 1.

Impossible event:-

An event that has no chance of occurring is called an impossible event.

$$P(E) = 0$$

Sure event:-

An event that has 100% probability of occurrence is called a sure event.

$$P(E) = 1$$

Complementary Events:- Complementary events are two outcomes of an event that are the only two possible outcomes. This is like flipping a coin and getting heads or tails.

$$P(E) + P(\bar{E}) = 1$$

DECK OF CARDS

Total 52 playing cards

4 suits - spade, Heart, Diamond and club

13 cards in each suit

4 Ace, 4 kings, 4 Queen & 4 Jack

My Top Ten

1. Two unbiased coins are tossed simultaneously. Find the probability of getting.
- (i) two heads
 - (ii) one tail
 - (iii) at most one head
 - (iv) one head
 - (v) at least one head
 - (vi) no head
- [CBSE 2011] [CBSE 2010, 2013]

Ans: (1) $1/4$, (2) $1/2$, (3) $1/2$
(4) $3/4$, (5) $3/4$, (6) $1/4$

2. Peter throws two different dice together and finds the product of the two numbers obtained. Reen throws a die and squares the number obtained. Who has the better chance to get the number 25? [CBSE 2017]

Ans: Reen has better chance

3. On the disc shown below, a player spins the arrow twice. The fraction $\frac{a}{b}$ is formed, where a is the number of the sector where the arrow stops after the first spin and b is the number of sectors where the arrow stops after the second spin. On every spin each of the numbered sectors has an equal probability of being the sector on which the arrow stops. What is the probability that the fraction $\frac{a}{b}$ is greater than 1? [CBSE 2016]

Ans: $5/12$

4. A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the numbers 1, 2, 3, ..., 12 as shown in fig. 16.3. What is the probability that it will point to:

- (i) 10?
- (ii) an odd number?
- (iii) a number which is multiple of 3?
- (iv) an even number?

Ans: (1) $1/12$, (2) $1/2$, (3) $1/3$, (4) $1/3$

5. All red face cards are removed from a pack of playing cards. The remaining cards are well shuffled and then a card is drawn at random from them. Find the probability that the drawn card is (i) a red card (ii) a face card and (iii) a card of clubs. [CBSE 2015]

Ans: (1) $10/23$, (2) $3/23$, (3) $13/96$

6. In a bag there are 44 identical cards with figure of circle or square on them. There are 24 circles, of which 9 are blue and rest are green and 20 square of which 11 are blue and rest are green. One card is drawn from the bag at random. Find the probability that it has the figure of (i) square (ii) green colour (iii) blue circle and (iv) green square. [CBSE 2015]

Ans: (1) $5/11$, (2) $6/11$, (3) $9/44$, (4) $9/44$

7. Red queens and all black jacks are removed from a pack of 52 playing cards. A card is drawn at random from the remaining cards, after reshuffling them. Find the probability that the card drawn is (i) a King (ii) of steel colour (iii) a face card (iv) a queen. [CBSE 2014]

Ans: (1) $1/12$, (2) $1/2$, (3) $1/6$, (4) $1/24$

8. Cards numbered from 11 to 60 are kept in a box. If a card is drawn at random from the box, find the probability that the number on the drawn card is.

Ans: (1) $1/2$, (2) $2/25$, (3) $1/5$, (4) $1/10$

- (i) an odd number

- (ii) a perfect square number

- (iii) divisible by 5

- (iv) a prime number less than 20

9. The King, queen and jack of clubs are removed from a deck of 52 playing cards and the remaining cards are shuffled. A card is drawn from the remaining cards. Find the probability of getting (i) a card of heart (ii) a queen (iii) a card of clubs (iv) a face card (v) a queen diamond. [CBSE 2009, 2011]

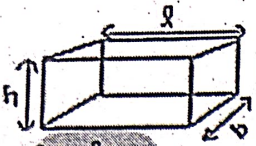
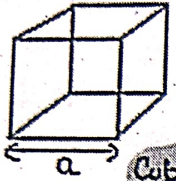
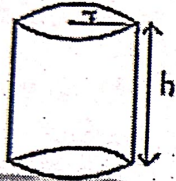
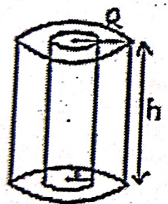
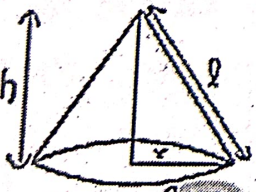
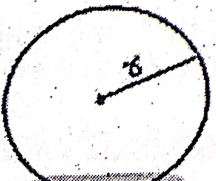

Ans: (1) $13/49$, (2) $3/49$, (3) $10/49$, (4) $9/49$, (5) $1/49$

10. A box contains 100 red cards, 200 yellow cards & 50 blue cards. If a card is drawn at random from the box, then find the probability that it will be (1) a blue card (2) not a yellow card (3) neither yellow nor a blue card.

CBSE 2012

Ans: (1) $1/7$, (2) $3/7$, (3) $2/7$

Surfaces Areas & Volume

FIGURES	CURVED/LATERAL SURFACE AREA	TOTAL SURFACE AREA	VOLUME	REMARKS
 <p>Cuboid</p>	$2(l+b)h$	$2(lb+lh+bh)$	$l \times b \times h$	l = length b = breadth h = height
 <p>Cube</p>	$4a^2$	$6a^2$	a^3	a = side of cube
 <p>Solid Cylinder</p>	$2\pi rh$	$2\pi rh + 2\pi r^2$	$\pi r^2 h$	r = radius of base h = height of cylinder
 <p>Hollow Cylinder</p>	External curved surface area $= 2\pi Rh$ Internal curved surface area $= 2\pi rh$	$2\pi Rh + 2\pi rh + 2\pi [R^2 - r^2]$	$\pi R^2 h - \pi r^2 h$	R = radius of outer base r = radius of inner base h = height of cylinder
 <p>Cone</p>	$\pi r l$	$\pi r^2 + \pi r l$	$\frac{1}{3} \pi r^2 h$	h = height of cone r = radius of base l = $\sqrt{h^2 + r^2}$ slant height
 <p>Sphere</p>	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3} \pi r^3$	r = radius of sphere
 <p>Hemisphere</p>	$2\pi r^2$	$2\pi r^2 + \pi r^2 = 3\pi r^2$	$\frac{2}{3} \pi r^3$	r = radius of hemisphere

My Top Ten

- The diameter of a metallic sphere is 6 cm. The sphere is melted and drawn into a wire of uniform cross-section. If the length of the wire is 36 m, find its radius. [CBSE 2013]
 Ans: 1mm
- Water is flowing at the rate of 5 km/hr through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Determine the time in which the level of the water in the tank will rise by 7 cm. [CBSE 2017]
 Ans: 2 hours
- A well with 10 m inside diameter is dug 14 m deep. Earth taken out of it is spread all around with to a width of 5 m to form an embankment. Find the height of embankment.
 Ans: 4.66 m
- A right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. The ice-cream is to be filled in cones of height 12 cm and diameter 6 cm having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream. [CBSE 2006C]
 Ans: 10
- From a solid circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and same base is removed. Find the volume of the remaining solid. Also, find the whole surface area. [CBSE 2009]
 Ans: $(156 + 12\sqrt{34})\pi$
- Rachel, an engineering student was asked to make a model in her workshop, which was shaped like a cylinder with two cones attached to its two ends, using thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. [NCERT]
 Ans: 66 cm^3
- A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass (use $\pi = 3.14$). [NCERT]
 Ans: 892.2624 kg
- A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm is diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3 . Check whether she is correct, taking the above as the inside measurements and $\pi = 3.16$. [NCERT]
 Ans: 346.5 cm^3
- A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter of the hemisphere can have? Find the total surface area of the solid. [NCERT]
 Ans: 332.5 cm^2
- A golf ball has diameter equal to 4.1 cm. Its surface has 150 dimples each of radius 2 mm. Calculate total surface area which is exposed to the surroundings assuming that the dimples are hemispherical.
 Ans: 71.68 cm^2

Statistics

MODE OF GROUPED DATA

$$\text{Mode} = J + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

- J = lower limit of modal class
- h = size of class interval
- f₁ = frequency of modal class
- f₀ = frequency of class preceding the modal class
- f₂ = frequency succeeding the modal class

$$\text{Median} = J + \frac{\frac{n}{2} - cf}{f} \times h$$

- J = lower limit of median class
- n = number of observation
- f = frequency
- cf = Cumulative frequency
- h = class size

Mean Direct Method

$$\bar{x} = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i}$$

Where $\sum f_i$ is the sum of observation from value $i=1$ to n and $\sum f_i x_i$ is number of observation from value $i=1$ to n

Assumed Mean Method

$$\bar{x} = a + \frac{\sum_{i=1}^n f_i d_i}{\sum_{i=1}^n f_i}$$

Where 'a' is assumed mean and d is deviation of 'a' from each of x_i also $d_i = x_i - a$

Step Deviation Method

$$\bar{x} = a + \frac{\sum_{i=1}^n f_i u_i}{\sum_{i=1}^n f_i} \times h$$

Where $u_i = \frac{x_i - a}{h}$ [h is the class size]

$$3 \text{ Median} = \text{Mode} + 2 \text{ Mean}$$

My Top Ten

1. Find the missing frequencies in the following frequency distribution if it is known as that the mean of the distribution is 50.

x:	10	30	50	70	90	
f:	17	f ₁	32	f ₂	19	Total 120.

Ans: 3

2. The mean of the following frequency table is 50. But the frequencies f₁ and f₂ in class 20-40 and 60-80 are missing. Find the missing frequencies.

class:	0-20	20-40	40-60	60-80	80-100	Total
frequency:	17	f ₁	32	f ₂	19	120

Ans: 28, 24

3. The daily income of a sample of 50 employees are tabulated as follows:

Income (in ₹):	1-200	201-400	401-600	601-800
No. of employees:	14	15	14	7

Ans: 11

Find the mean daily income of employees. [NCERT EXEMPLAR]

4. If the mean of the following distributions is 51, find the value of p:

class:	0-20	20-40	40-60	60-80	80-100
frequency:	7	p	10	9	13

[CBSE 2006 (I)] Ans: 28, 24

5. Find the missing frequencies in the following distribution, if the sum of all the frequencies is 120 and the mean is 50.

class:	0-20	20-40	40-60	60-80	80-100
frequency:	17	f ₁	32	f ₂	19

Ans: 28, 24

[NCERT EXEMPLAR]

6. Find the value of x, if the mode of the following data is 25:

15, 20, 25, 18, 14, 15, 25, 15, 18, 16, 20, 25, 20, x, 18. Ans: 356.5

7. Calculating the missing frequency from the following distribution, it being given that the median of the distribution is 24:

Age in years:	0-10	10-20	20-30	30-40	40-50
No. of persons:	5	25	?	18	7

Ans: 8, 7

8. Find the mean, median and mode of the following data:

Classes:	0-20	20-40	40-60	60-80	80-100	100-120	120-140
Frequency:	6	8	10	12	36	5	3

[CBSE 2009] Ans: 25

9. Find the value of p, if the mean of the following distribution is 7.5.

x:	3	5	7	9	11	13
f:	6	8	15	p	8	4

Ans: 25

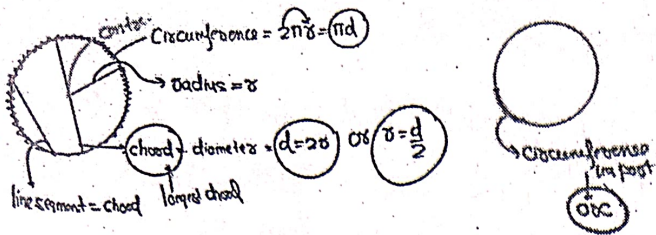
10. If the median of the distribution given below is 28.5, find the value of x and y.

Class interval:	0-10	10-20	20-30	30-40	40-50	50-60	Total 60
No. of students:	5	x	20	15	y	5	

Ans: median = 61.66, mean = 62.4, mode = 65

[NCERT]

Circles

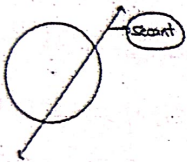


Definition - A circle is a collection of all those points in a plane which are at **fixed distance** from a **fixed point**.

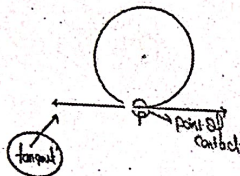
radius

centre

Secant



Tangent



SECANT -

A line which intersects a circle in two distinct points is called a secant of the circle.

TANGENT

- ✓ A tangent to a circle in exactly one point.
- ✓ The point is called the point of contact of the tangent and the line is said to touch the circle at this point.
- ✓ The word tangent is originated from the Latin word TANGERE which means "to touch".

Note:

The point of contact is the only point which is common to the tangent and the circle and every other point on the tangent lies outside the circles. Thus, of all the points on a tangent to a circle, the point of contact is nearest to the center of the circle.

THEOREM 1

A tangent to a circle is perpendicular to the radius through the point of contact. CBSE 2009, 12, 14, 15, 16

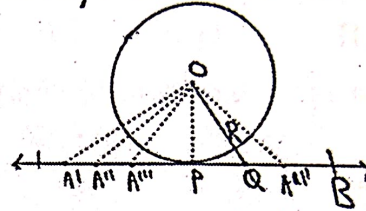
Q: A C (O, r) and a tangent AB, point of point 'P'
TOP: OPLAB

Proof: $OB > OR$ (R lies outside the circle)

$$OR = OP$$

$$OB > OP$$

similarly, $OA', OA'', OA''', OA'''' >>> OP$

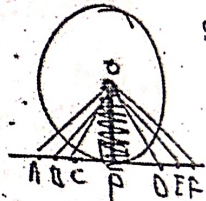


Any point and its distance from centre 'O' other than 'P' will be greater than OP. 'OP' is the shortest distance b/w 'O' and AB. also, shortest distance is \perp distance

$OP \perp AB$

shortest distance is always perpendicular

shortest distance $\rightarrow OP$



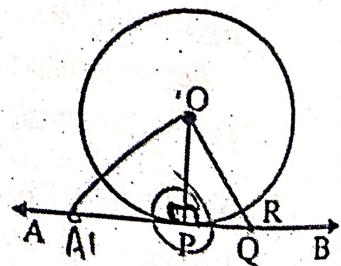
THEOREM-2

A line drawn through the end point of a radius and perpendicular to it is a tangent to the circle. CBSE 2012, 13

tangent prove karna hai

ahhi point pay touch

- 1) shortest distance = OP (reason $\rightarrow \perp$ distance)
- 2) OP radius hai



Given: A radius OP of a circle C(O, r) and a line APB, perpendicular to OP.

Prove: AB is a tangent to the circle at the point P.
Proof: Take a point Q, different from P, on the line AB.

Now, $OP \perp AB$

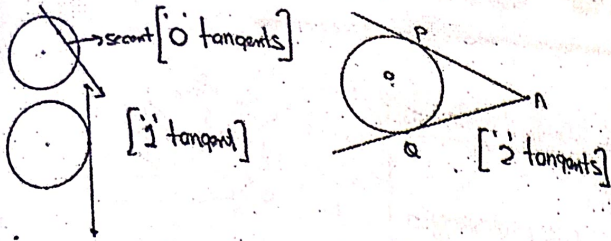
Among all the line segments joining O to a point on AB, OP is the shortest.

$OQ > OP$ and 'OP' is radius
Q lies outside the circle.

Thus every point on AB, other than P, lies outside the circle. This shows that AB meets the circle only at the point P.

Hence AB is a tangent to the circle at P

Tangents from a point to a circle



These facts can be summarized as follows

- (i) No tangent can be drawn to a circle from a point lying inside it.
- (ii) One and only one tangent can be drawn to a circle at a point on the circle.
- (iii) Two tangents can be drawn to a circle from a point lying outside it.

THEOREM 3

The lengths of two tangents drawn from an external point to a circle are equal

← BSE 2008, 09, 10, 13, 14, 15, 19, 17, 18

GI: AP and AB are tangents to a circle.

$$AP = AB$$

In $\triangle APO$ and $\triangle ABO$

$$OB = OP \text{ (radius)}$$

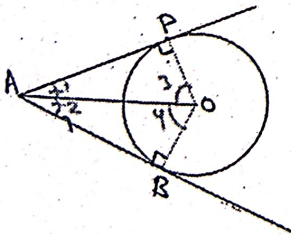
$$\angle OPA = \angle OBA (90^\circ) \text{ [tangent is } \perp \text{ to radius]}$$

$$OA = OA \text{ (common)}$$

By RHS, $\triangle APO \cong \triangle ABO$

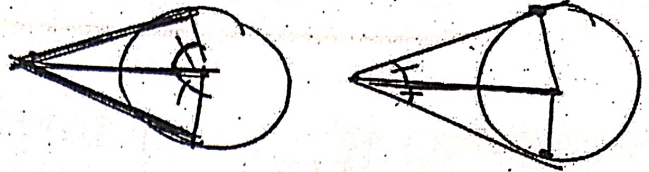
By CPCT $AP = AB$

Also $\angle 1 = \angle 2$
 $\angle 3 = \angle 4$



THEOREM 4

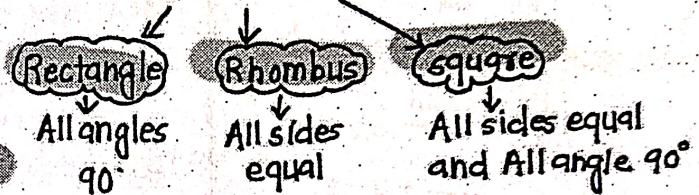
If two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre, joining the centre to that point i.e. the centre lies on the bisector of the angle between the tangents.



//gm

- ✓ opp. sides //
- ✓ opp. side equal
- ✓ opp. angles equal
- ✓ diagonal's bisect each other

//am



Area Related to Circles

• Area of circle = πr^2
 $\pi \approx \text{rotational} \rightarrow 3.14$
 $\frac{22}{7}$



• $\frac{\pi r^2}{2} = \frac{1}{2} \pi r^2$

• $\frac{\pi r^2}{4}$ → Quadrant

• circumference of circle
 $= 2\pi r$

• perimeter semicircle

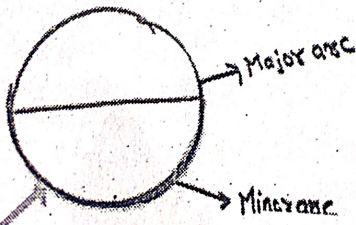
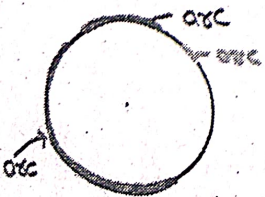
$= \frac{2\pi r}{2} + r + r$
 $= \pi r + 2r$

$= \frac{2\pi r + r + r}{2}$
 $= \frac{\pi r + 2r}{2}$

Major and Minor Arc :-

An arc of a circle whose length is less than that of a semi circle of the same circle is called a minor Arc.

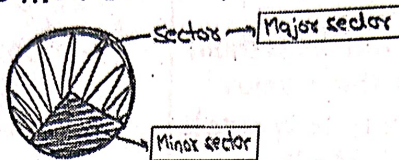
An arc of a circle whose length is greater than that of a semi circle is called a major arc.



Sector of a circle and its Area

Sector of a circle

The part of the circular region enclosed by two radii and corresponding arc is called sector of a circle.

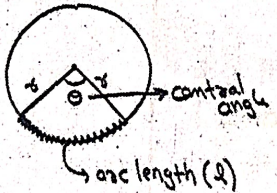


Minor sector -

A sector of a circle is called a minor sector, if the minor arc of the circle is a part boundary.

Major Sector -

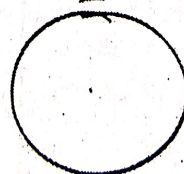
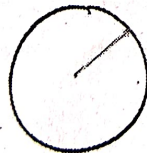
A sector of a circle is called a major sector, if the major arc of the circle is a part of its boundary.



$l = \frac{\theta}{360} \times 2\pi r$

Area of sector = $\frac{\theta}{360} \times \pi r^2$

Area of sector = $\frac{1}{2} \times l \times r$



$360^\circ = \pi r^2$

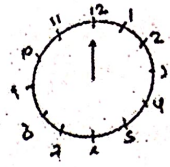
$1^\circ = \frac{\pi r^2}{360^\circ}$

$\theta = \frac{\theta}{360} \times \pi r^2$

$360^\circ = 2\pi r$

$1^\circ = \frac{2\pi r}{360^\circ}$

$\theta = \frac{\theta}{360} \times 2\pi r$

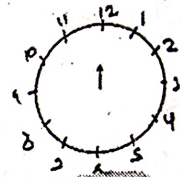


Minute Method

$60 \text{ min} = 360^\circ$

$1 \text{ min} = \frac{360}{60}$

$1 \text{ min} = 6^\circ$



Hours hand

$12 \text{ hr} = 360^\circ$

$1 \text{ hr} = \frac{360}{12}$

$1 \text{ hr} = 30^\circ$